

Rectangular Apartment

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First, let us assume that the turtle's apartment is infinite and contains no furniture. Consider which cells the turtle will visit if it starts its path from cell $(1, 1)$. If the turtle eventually shifts to the right by more than m , then the answer to the problem is 0. Otherwise, for each y from 1 to m , it is easy to see that the turtle either visits no cells in column y , or visits a segment of cells $[l_y, r_y]$: that is, all cells (x, y) where $l_y \leq x \leq r_y$.

The values l_y and r_y can be precomputed in $\mathcal{O}(|s|)$. Using these values, it is then easy to check in $\mathcal{O}(m)$ for a fixed cell (i, j) whether it satisfies the condition of the problem:

- For each y from 1 to m , if the segment $[l_y, r_y]$ is non-empty, one must verify that $j + y - 1 \leq m$, and also that all cells in this column on the segment $[i + l_y - 1, i + r_y - 1]$ exist and are not occupied by furniture.

To perform this check efficiently for each column, it is sufficient to compute prefix sums in every column. This method allows checking a fixed cell in $\mathcal{O}(m)$, which gives a solution in $\mathcal{O}(|s| + nm^2)$ if we perform the check independently for every cell. This solution is sufficient to solve the problem for full score.

This problem can also be solved in $\mathcal{O}(|s| + nm \log(nm))$ using multiplication of two-dimensional polynomials, but this was not required.