

# Permutations and Queries

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Let us represent a permutation as a set of points  $(i, p_i)$ . Then the first type of operation transforms all points  $(x, y)$  into  $(n - x + 1, y)$ . The second type of operation transforms  $(x, y)$  into  $(x, n - y + 1)$ . The third type of operation transforms  $(x, y)$  into  $(y, x)$ .

Notice that using any number of these operations, we can obtain no more than eight different permutations, which we describe as follows. Let  $f_1, f_2, f_3$  be three variables that take values 0 or 1.

Let  $p$  be our initial permutation, and then we apply the following modifications to it in order:

1. If  $f_1 = 1$ , then we perform the third type of operation (that is, we transform  $(x, y)$  into  $(y, x)$ ).
2. If  $f_2 = 1$ , then we perform the first type of operation (that is, we transform  $(x, y)$  into  $(n - x + 1, y)$ ).
3. If  $f_3 = 1$ , then we perform the second type of operation (that is, we transform  $(x, y)$  into  $(x, n - y + 1)$ ).

Initially, we precompute the cost of the eight permutations corresponding to every possible set of variables  $f_1, f_2, f_3$ .

Now we maintain the current variables  $f_1, f_2, f_3$ , which are initially equal to zero.

- For a query of the first type, we replace  $f_2$  with  $1 - f_2$ .
- For a query of the second type, we replace  $f_3$  with  $1 - f_3$ .
- For a query of the third type, we replace  $f_1$  with  $1 - f_1$  and swap  $f_2$  and  $f_3$ .

Thus, after each query, we maintain the current variables  $f_1, f_2, f_3$  and output the precomputed cost for such flags. The running time is  $O(n \log n + q)$ , where the  $\log$  comes from binary exponentiation used to compute the cost.